PRACTICE PAPER -2 (2020-21)

CLASS XII

MATHEMATICS

TIME ALLOWED: 3 HOURS **MAXIMUM MARKS: 80**

GENERAL INSTRUCTIONS:

1. This question paper contains two parts A and B. Each part is compulsory. Part A carries 24 marks and Part B carries 56 marks.
2. Part-A has Objective Type Questions and Part -B has Descriptive Type Questions.
3. Both Part A and Part B have choices.

Part – A

1. It consists of two sections- I and II.
2. Section I comprises of 16 very short answers type questions.
3. Section II contains 2 case studies. Each case study comprises of 5 case-based MCQs. An examinee is to attempt any 4 out of 5 MCQs.

Part – B

1. It consists of three sections- III, IV and V.
2. Section III comprises of 10 questions of 2 marks each.
3. Section IV comprises of 7 questions of 3 marks each.
4. Section V comprises of 3 questions of 5 marks each.
5. Internal choice is provided in 3 questions of Section –III, 2 questions of Section IV and 3 questions of Section-V. You have to attempt only one of the alternatives in all such questions.

PART – A

SECTION – I

(All questions are compulsory. In case of internal choices attempt any one)

1. Write the smallest reflexive relation on set A = {1, 2, 3, 4}.

Or

If f : A $\rightarrow$ B is an injection such that range of f ={a}. Determine the number of elements in A.

2. If R is a symmetric relation on a set A, then write a relation between R and R^{-1}.

3. Let A = {1, 2, 3}. Then,what is the number of equivalence relations containing (1, 2) ?

Or

If A = {a, b, c} and B = {- 2, -1,0,1,2}, write total number of one-one functions from A to B.

4. If $\begin{bmatrix} a+b & 2 \\ 5 & ab \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 5 & 8 \end{bmatrix}$, find the values of a and b.

5. If I is the identity matrix and A is a square matrix such that $A^2 = A$, then what is the value of $(I + A)^2 - 3A$?

Or

Write a square matrix which is both symmetric as well as skew-symmetric.

6. A matrix A of order 3 x 3 is such that | A | = 4. Find the value of | 2A |.

7. Write a value of $\int e^{x}\sec x(1+\tan x)dx$

Or

Write a value of $\int \frac{1-\sin x}{\cos^{2} x}dx$

8. Find the area bounded by the curves y = sin x between the ordinates x = 0, x = π and the x –axis.

9. If sin x is an integrating factor of the differential equation $\frac{dy}{dx}$ + Py = Q, then write the value of P.

Or

Write the order of the differential equation associated with the primitive

$y=C_1+C_2 e^{x}+C_3 e^{-2x+C_4}$, where C_1, C_2, C_3, C_4 are arbitrary constants.

10. Find a vector in the direction of $\vec{a}=2\hat{i}-\hat{j}+2\hat{k}$, which has magnitude of 6 units.

11. For what value of λ are the vectors $\vec{a}$ = 2i + λi + k and $\vec{b}$ = I – 2j + 3k perpendicular to each other?

12. If $\vec{a}$ and $\vec{b}$ are mutually perpendicular unit vectors, write the value of $|\vec{a}+\vec{b}|$.

13. The cartesian equation of a line AB is $\frac{2x-1}{\sqrt{3}}=\frac{y+2}{2}=\frac{z-3}{3}$. Find the direction cosines of a line parallel to AB.

14. Write the distance between the parallel planes 2x - y + 3z = 4 and 2x - y + 3z = 18.

15. If P (A) = 0.3, P (B) = 0.6, P (B/A) = 0.5, find P (A $\cup$ B).

16. If X is a random-variable with probability distribution as given below :

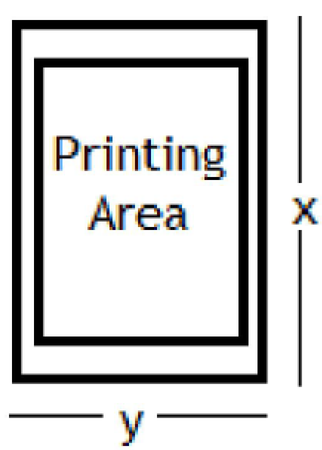
X:	0	1	2	3
P(X = x):	k	3k	3k	k

Find the value of k .

SECTION II

(Both the Case study based questions are compulsory. Attempt any 4 sub parts from each question 17 and 18 . Each part carries 1 mark)

17. Following is the pictorial description for a page.



The total area of the page is 150 cm^2. The combined width of the margin at the top and bottom is 3 cm and the side 2 cm.
Using the information given above, answer the following :

(i) The relation between x and y is given by
(a) (x -3)y =150
(b) xy =150

(c) x(y - 2) =150
(d) (x - 2)(y - 3) =150

(ii) The area of page where printing can be done, is given by
(a) xy
(b) (x + 3)(y + 2)
(c) (x -3)(y - 2)
(d) (x -3)(y + 2)

(iii) The area of the printable region of the page, in terms of x, is
(a) 156 + 2x + 450/x
(b) 156 - 2x + 450/x
(c) 156 - 2x – 45/x
(d) 156 - 2x – 450/x

(iv) For what value of 'x', the printable area of the page is maximum?
(a) 15 cm
(b) 10 cm
(c) 12 cm
(d) 15 units

(v) What should be dimension of the page so that it has maximum area to be printed?
(a) Length =1 cm, width = 15 cm
(b) Length = 15 cm, width = 10 cm
(c) Length = 15 cm, width = 12 cm
(d) Length = 150 cm, width = 1 cm

18. Let X denotes the no. of colleges where you will apply after your results and P(X = x) denotes your probability of getting admission in x number of colleges. It is given that

$$P(X=x)=\begin{cases} kx, \text{ if } x=0 \text{ or } 1 \\ 2kx, \text{ if } x=2 \\ k(5-x), \text{ if } x=3 \text{ or } 4 \\ 0, \text{ if } x>4 \end{cases}$$

; where k is a positive constant.

Based on the above information answer the following :

(i) The value of k is
(a) 1
(b) 1/3
(c) 1/7
(d) 1/8

(ii) The probability that you will get admission in exactly one college, is
(a) 1/2
(b) 1/3
(c) 1/8
(d) 1/5

(iii) The probability that you will get admission in at most two colleges, is
(a) 7/12
(b) 5/8
(c) 5/21
(d) 8/17

(iv) What is the probability that you will get admission in at least 2 colleges?

(a) 1/3
(b) 2/7
(c) 3/8
(d) 7/8

(v) What is the probability that you will get admission in more than 4 colleges?
(a) 0
(b) 1
(c) 1/2
(d) 1/8

PART – B
SECTION III

19. If $\frac{dy}{dx} = e^{-2y}$ and $y = 0$ when $x = 5$, then the value of x when $y = 3$.

20. If the points $A(-1, 3, 2)$, $B(-4, 2, -2)$ and $C(5, 5, \lambda)$ are collinear, find the value of λ.

21. Find a vector in the direction of $\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$ which has magnitude 6 units.

22. Find the principal value of

$$\tan^{-1}\left(\tan\frac{7\pi}{6}\right) + \cot^{-1}\left(\cot\frac{7\pi}{6}\right).$$

23. Evaluate

$$\int_0^{\frac{\pi}{2}} \log\left(\frac{3+5\cos x}{3+5\sin x}\right) dx.$$

OR Evaluate $\int \frac{\log(\sin x)}{\tan x} dx$

24. Find the slope of the normal to the curve $x = a\cos^3\theta$, $y = a\sin^3\theta$ at $\theta = \frac{\pi}{4}$.

25. If $y = \tan^{-1}\left(\frac{\cos x}{1+\sin x}\right)$ find $\frac{dy}{dx}$

26. Find matrices X and Y, if

$$X + Y = \begin{bmatrix} 5 & 2 \\ 0 & 9 \end{bmatrix} \quad \text{and} \quad X - Y = \begin{bmatrix} 3 & 6 \\ 0 & -1 \end{bmatrix}$$

OR Find the value of k so that the points $A(5, 5)$ $B(k, 1)$ and $C(11, 7)$ are collinear.

27. A bag contains 15 tickets numbered from 1 to 15. A ticket is drawn and then another ticket is drawn without replacement. Find the probability that both tickets will show even numbers.

OR A die is tossed thrice. Find the probability of getting an odd number at least once.

28. Using integration, find the area of the region bounded by the line $y - 1 = x$, the x-axis and the ordinates $x = -2$ and $x = 3$.

SECTION IV
(All questions are compulsory. In case of internal choices attempt any one.)

29. Let Z be the set of all integers and R be the relation on Z defined as $R = \{(a, b) : a, b \in Z, (a - b)$ is divisible by $5\}$. Prove that R is an equivalence relation.

30. Using integration, find the area of the region bounded by the following curves, after making a rough sketch:

$$y = 1 + |x + 1|, \quad x = -3, \quad x = 3, \quad y = 0.$$

OR Find the area bounded by the parabola $y^2 = 4x$ and the straight line $x + y = 3$.

31. Evaluate

$$\int_0^{\pi} \frac{x \sin x}{1 + \cos^2 x} dx$$

32. Find the intervals in which $f(x) = (x + 1)^3 (x - 3)^3$ is increasing or decreasing.

33. If $x = \tan\left(\frac{1}{a}\log y\right)$, show that $(1 + x^2)\frac{d^2y}{dx^2} + (2x - a)\frac{dy}{dx} = 0$.

34. If $x = ae^{\theta}(\sin\theta - \cos\theta)$ and $y = ae^{\theta}(\sin\theta + \cos\theta)$, find $\frac{dy}{dx}$

OR If $y^x = e^{y-x}$, prove that $\frac{dy}{dx} = \frac{(1 + \log y)^2}{\log y}$

35. Solve the following differential equation

$$(x^2 - 1)\frac{dy}{dx} + 2xy = \frac{1}{x^2 - 1}; \ |x| \neq 1$$

SECTION V

(All questions are compulsory. In case of internal choices attempt any one.)

36. Find the equation of the plane through the intersection of the planes $\vec{r} \cdot (\hat{i} + 3\hat{j}) - 6 = 0$ and $\vec{r} \cdot (3\hat{i} - \hat{j} - 4\hat{k}) = 0$, whose perpendicular distance from origin is unity.

OR

Find the distance of the point (2, 3, 4) from the line $\frac{x+3}{3} = \frac{y-2}{6} = \frac{z}{2}$ measured parallel to the plane $3x + 2y + 2z - 5 = 0$.

37. Solve the following LPP Graphically :

Maximise $z = 1000x + 600y$

Subject to $x + y \leq 200, x \geq 20, y \geq 4x, x, y \geq 0.$

OR

Solve the following linear programming problem graphically:

Maximise $z = 6x + 5y$ subject to $3x + 5y \leq 15, 5x + 2y \leq 10, x, y \geq 0.$

36. Solve the following system of equations $3x + 2y + z = 6; 4x - y + 2z = 5; 7x + 3y - 3z = 7$.

If $A = \begin{bmatrix} 3 & 2 & 1 \\ 4 & -1 & 2 \\ 7 & 3 & -3 \end{bmatrix}$ find A^{-1}.

OR

Find AB, use this to solve the system of equations $x - y = 3$, $2x + 3y + 4z = 17$, $y + 2z = 7$.

Where $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ **and** $B = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$.